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Chiral QCD phase transition in nuclear collisions

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Abstract. The characteristic aspects of multiparticle states generated at $T = T_c$, as a result of chiral QCD phase transition, are studied in the framework of the O(4) ϕ^4 theory. Predictions concerning critical events, in connection with current and future experiments with ultrarelativistic heavy ions, are presented.

1 Introduction

In ultrarelativistic nuclear collisions, A+A, one expects to detect new processes related to QCD phase transitions (deconfinement, chiral) in a class of (critical) events associated with quark-gluon plasma (QGP) formation. With particular reference to the chiral transition in QCD there is growing evidence, mainly from lattice considerations, that it is of second order [1]. According to a general argument by Wilczek [2] the universality class for this second order transition is represented by the O(4) linear σ-model. In view of the extensive experimental programme on heavy ions now in progress (Pb-SPS, RHIC, LHC) the effort to derive detectable predictions based on the O(4) ϕ^4 theory of chiral QCD phase transition is of high priority [3,4].

On the experimental side, the crucial issue is to relate the role of QCD chiral dynamics to multihadron production patterns as these occur in heavy ion collisions. In the present work we attempt a step in this direction based on the assumption that, in these experiments, the conditions required to consider the hadronization process as a static phase transition are well fulfilled. We therefore adopt a space-time evolution picture, for the system (QGP) produced in the central rapidity region, which favors the persistence of local equilibrium conditions. A standard account for such a state of affairs is provided by Bjorken's inside-outside scheme for the development of the process [5].

Our efforts will focus on a local system defined on the critical isotherm, $T = T_c$, pertaining to the chiral transition. The relevant critical exponent δ is associated with the fractal dimension of the (critical) local system at $T = T_c$ and, therefore, with a particular intermittency pattern, in momentum space, of the hadrons produced from the local source [3].

The paper is organized as follows: in Sect. 2 we derive a Ginzburg-Landau (GL) free energy for the hadronic system at the critical temperature based on the $O(4)$ Heisenberg model as a microscopic action for the QCD chiral dynamics. In Sect. 3 we use the GL free energy derived in 2 to study the thermodynamics of the chiral condensates, considered as local hadronization sources formed at the critical temperature, and a number of phenomenological characteristics related to the hadron production from the local sources are presented. Finally in Sect. 4 we give our conclusions and a brief outlook concerning the extension of the local hadronization picture to a global description expected to describe the multihadron production at the real events in ultrarelativistic heavy ion collisions.

2 Critical fluctuations

In our approach the actual contact between QCD and the O(4) Heisenberg model occurs at the critical point. The question arises as to which side of this critical point the two theories describe physical systems that are compatible with each other. Consider the O(4) model. At microscopic level it corresponds to a theory of scalar quantum fields whose particle content is identified with pions, plus an additional radial mode (σ-field). From such a microscopic standpoint suppose we embark on a procedure through which we integrate out those degrees of freedom that are progressively encountered from very short distances up to scales where a *macroscopic* profile of the fields, including fluctuations, begins to emerge. In the limit where all degrees of freedom (of the microscopic theory) have been integrated out, the so called effective action for the system is obtained which accounts for configurations of the macroscopic field ϕ_{class}.

Adding finite temperature, via the standard procedure, one finally arrives at a bonafied thermodynamical description of the system which, in the present case and near the critical temperature, can be characterized as "thermodynamics of chiral condensates".

It follows from the above discussion that the physical relevance of the O(4) Heisenberg model, viewed as a microscopic theory, refers to the hadronic side of QCD. To the extent, now, that the freeze-out temperature T_f at which one meets the asymptotic states, is very close to the critical temperature ($T_c \approx T_f$), as expected in a transition of second order the overlap of the two theoretical models *occurs in the immediate vicinity of the critical point at the hadronic side*.

In a finite temperature context, the effective action describes a three-dimensional field theory which exhibits thermal fluctuations. On the critical surface, we write:

$$\Gamma_c\left[\phi\right] = \beta_c \int d^3x \left[\frac{1}{2}\left(\partial\phi\right)^2 + \ U_c\left(\phi\right)\right] \tag{1}$$

with the fields entering the above relation being classical, in the sense described earlier (the subscript "class" has been dropped for notational economy). The O(4) effective potential $U_c\left(\phi\right)$ at $T = T_c$ reads [7]:

$$U_c\left(\phi\right) \approx g_c \beta_c^{2(k-2)} \left(\frac{\phi^2}{2}\right)^k , \qquad k = \frac{3}{1+\eta} \tag{2}$$

where $g_c \approx 6$, and $\eta \approx 0.034$. The last parameter is the anomalous dimension assigned to the renormalized scalar field ϕ. It accounts for the absorption of all (quantum) fluctuation modes as they are being integrated out [6]. Note that (1) and (2) take the global view of a single inertial observer. In particular, they result via the imposition of periodic boundary conditions between two flat 3-dimensional hypersurfaces, corresponding to the Minkowskian equal time planes $x_o = 0$ and $x_o = i\beta_c$. For the problem in hand, the inside-outside cascade evolution picture calls for a monitoring of the system by a collection of inertial observers each of whom contributes to the local description of the system. Putting their observations together leads us to the construction of thermodynamical quantities related to the entire system as functions of the proper time.

We proceed to adapt $\Gamma_c\left[\phi\right]$ to the dynamical evolution picture we have just described by restricting ourselves to the domain of jurisdiction of a single (inertial) observer. For this purpose we introduce rapidity and proper time coordinates (ξ, τ) so that the longitudinal space element corresponding to a local observer, in a hadronization point $\xi = \xi_o$, becomes $dx_{\parallel} = \tau\cosh(\xi - \xi_o)d\xi$. With this choice and for the purpose of describing the system at $T = T_c$, a longitudinal integration along the critical hyperbola $\tau = \tau_c$ must be performed. We thereby obtain the following integral representation for the effective action pertaining to a local description of the system, as viewed by the observer at $\xi = \xi_o$:

$$\begin{aligned}\Gamma_c\left[\phi\right] = \beta_c\tau_c \int_{\Delta} d\xi \int_{S_\perp} d^2\mathbf{x}_\perp \Bigg[\frac{1}{2\tau_c^2\cosh(\xi-\xi_o)}\left(\frac{\partial\phi}{\partial\xi}\right)^2 \\ + \cosh(\xi-\xi_o)\left(\frac{1}{2}\left(\nabla_\perp\phi\right)^2 + U_c\left(\phi\right)\right)\Bigg]\end{aligned} \tag{3}$$

where Δ is the rapidity size and $S_\perp$ the transverse area of the system.

Having stationed ourselves near the critical point we view (3) as a Ginzburg-Landau (GL) free energy which governs the behaviour of the hadronic system in a local mode of description. Our interpretation is in accordance with the GL description of the Feynman-Wilson fluid near the critical temperature, first introduced by Scalapino and Sugar [8] and subsequently developed by many others [9].

3 Thermodynamics of chiral condensates

In order to reveal the thermal behaviour of chiral condensates we recall that the 4-component classical field $\phi_i(\mathbf{x}_\perp, \xi)$; $i = 1, 2, ..4$, describes coherent emission of pions and sigmas at $T = T_c$ from a hadronizing local source. To this end, we place the center of the source at the point $\xi = \xi_o$, $\mathbf{x}_\perp = 0$, which are the coordinates of the local observer. Furthermore, we set, for simplicity, $\xi_o = 0$.

To facilitate our discussion we consider, following [8], a density operator $\hat{\rho}$ which is diagonal in the O(4) coherent-state representation and corresponds to the free energy (3). In this context one may summarize the basic equations as follows:

$$\begin{aligned}\hat{\rho} &= \frac{1}{Q}\int \left[\delta\phi_i\right] |\phi_1..\phi_4\rangle e^{-\Gamma_c[\phi]} \langle\phi_1..\phi_4| \\ Q &= \int \left[\delta\phi_i\right] e^{-\Gamma_c[\phi]} \\ \hat{a}_i(\mathbf{x}_\perp, \xi)|\phi_1..\phi_4\rangle &= \phi_i(\mathbf{x}_\perp, \xi)|\phi_1..\phi_4\rangle \\ \left[\hat{a}_i(\mathbf{x}_\perp, \xi), \hat{a}_i^+(\mathbf{x}'_\perp, \xi')\right] &= \delta^{(2)}(\mathbf{x}_\perp - \mathbf{x}'_\perp)\delta(\xi - \xi')\end{aligned} \tag{4}$$

In (4) $|\phi_1..\phi_4\rangle$ are O(4) coherent states and $\hat{a}_i(\mathbf{x}_\perp, \xi)$ annihilation operators associated with the particle content of the field $\phi_i(\mathbf{x}_\perp, \xi)$. It is of interest to note that these states, corresponding to real eigenfields of the annihilation operators, form only a subspace of the coherent hadronic world. They define a subsystem of pions and sigmas, the Statistical Mechanics of which is described by the diagonal density matrix $\hat{\rho}$ (4). The remaining eigenstates of $\hat{a}_i(\mathbf{x}_\perp, \xi)$ corresponding, in general, to complex eigenfields are dissipated out and are associated with conventional (noncritical) events which form the thermal environment of the critical system under investigation.

The multiplicity operator is written:

$$\hat{\mathrm{n}} = \sum_{i=1}^{4} \int d^2\mathbf{x}_\perp d\xi \hat{a}_i^+(\mathbf{x}_\perp, \xi)\hat{a}_i(\mathbf{x}_\perp, \xi) \tag{5}$$

and the average multiplicity of hadrons (pions and sigmas), $<\mathrm{n}> = \mathrm{tr}(\hat{\mathrm{n}}\hat{\rho})$, becomes:

$$<\mathrm{n}> = \frac{1}{Q}\int \left[\delta\phi_i\right] \left(\int d^2\mathbf{x}_\perp d\xi \phi^2(\mathbf{x}_\perp, \xi)\right) e^{-\Gamma_c[\phi]} \tag{6}$$

It follows that one may interpret $\phi^2(\mathbf{x}_\perp, \xi) = \sum_{i=1}^{4}\phi_i^2(\mathbf{x}_\perp, \xi)$ as the density of hadrons in 3-d space $(\mathbf{x}_\perp, \xi)$, associated with a local source of hadronization, chosen at the point $\mathbf{x}_\perp = 0, \xi = 0$. This last remark implies that, relevant to the search for macroscopic observables within the framework of our description, the thermal average

$< \phi^2(\mathbf{x}_\perp, \xi) >$ in fact represents the density-density correlation in this space.

Having reached the asymptotic states of the hadronic system in a smooth manner with the aid of a collection of local sources of hadronization, we are now ready to construct the partition function. More specifically, the local observers are assigned respective subdivisions $\mathrm{V}(\mathrm{m}_1, \mathrm{m}_2) = \frac{\mathrm{V}}{\mathrm{m}_1 \mathrm{m}_2^2}$ of the total cylindrical volume $\mathrm{V} = \pi \mathrm{R}^2_{\perp,c} \Delta$ ($\Delta \to \frac{\Delta}{\mathrm{m}_1}, \mathrm{R}_{\perp,c} \to \frac{\mathrm{R}_{\perp,c}}{\mathrm{m}_2}$; $\mathrm{m}_1, \mathrm{m}_2 \geq 1$) within which corresponding multiplicities (n) are uniformly distributed. Identifying $\phi^2 = \frac{\mathrm{n} \mathrm{m}_1 \mathrm{m}_2^2}{\mathrm{V}}$ (density of hadrons in the cylindrical volume $\mathrm{V}(\mathrm{m}_1, \mathrm{m}_2)$ emitted from the source at the origin), (3)–(4) lead to the following canonical partition function for the *hadronic fluid* at $T = T_c$:

$$Z(n, \mathrm{V}, T_c) = \sum_{m_1, m_2 \geq 1} \exp\left\{ -\frac{c_A g_c \beta_c^{2k-2}}{2^k \mathrm{V}^{k-1}} \times \mathrm{n}^k \mathrm{m}_1^{k-1} \mathrm{m}_2^{2k-2} f\left(\frac{\Delta}{m_1}\right) \right\} \quad (7)$$

where $f(\chi) \equiv \frac{2\sinh(\frac{\chi}{2})}{\chi}$.

For a typical size of the local system $\Delta \leq 1$, we may safely put, in what follows, $f(\frac{\Delta}{m_1}) \approx 1$. In (7) we have introduced the dimensionless quantity $c_A = \frac{\tau_c}{\beta_c}$ which gives a measure of the hadronization time-scale. In the standard description of the nuclear A + A collision process the A-dependence of this parameter suggests $c_A \approx A^{\frac{1}{3}}$. The volume $\mathrm{V} = \pi \mathrm{R}^2_{\perp,c} \Delta$, available at $T = T_c$, depends on the transverse radius $\mathrm{R}_{\perp,c}$, a characteristic parameter in the critical system that may reach values much higher than the geometrical size of the original nuclei. Moreover, as we read from (7), a characteristic volume scale $\mathrm{V}_0 = \beta_c^2 \left(\frac{c_A g_c}{2^k}\right)^{\frac{1}{k-1}}$ introduces itself, which, for the actual values of the parameters ($\beta_c \approx \mathrm{m}_\pi^{-1}$, $g_c \approx 6$, $k \approx 3$), is of the order of $1.7 A^{\frac{1}{6}}$ (fm^2). This is a sufficiently small scale compared to the volume available in A + A collisions a circumstance which guarantees that in experiments with heavy ions the thermodynamic limit ($\mathrm{V} \gg \mathrm{V}_0$) can be easily reached. As a consequence, the critical fluctuations of the system, which are expected to develop in the thermodynamic limit, may become visible in these collisions.

The precise nature of the density fluctuations at $T = T_c$ may be revealed by studying the multiplicity moments $< \mathrm{n}^q; a, b >$ ($q = 1, 2, ..$) within cylindrical domains of volume $\mathrm{V}_{ab} = \frac{\mathrm{V}}{ab^2}$ ($a \geq 1, b \geq 1$). In particular for $a \gg 1$ or $b \gg 1$ one may search for fractal structures both in rapidity and transverse space. We have:

$$\langle \mathrm{n}^q; a, b \rangle = \frac{\pi^2}{Q(a,b)} \sum_{n, m_1, m_2} \mathrm{n}^{q+1} \times \exp\left\{ -\alpha^{k-1} \mathrm{n}^k \mathrm{m}_1^{k-1} \mathrm{m}_2^{2k-2} \right\} \quad (8)$$

where $\alpha = \frac{\mathrm{V}_0}{\mathrm{V}}$ and

$$Q(a,b) = \pi^2 \sum_{n, m_1, m_2} \mathrm{n} \exp\left\{ -\alpha^{k-1} \mathrm{n}^k \mathrm{m}_1^{k-1} \mathrm{m}_2^{2k-2} \right\} \times (\mathrm{n} \geq 1, \mathrm{m}_1 \geq a, \mathrm{m}_2 \geq b)$$

Introducing the scaling variable $\mathrm{z} = \frac{\mathrm{V}_0}{\mathrm{V}_{ab}}$ we find

$$\langle \mathrm{n}^q; a, b \rangle = \mathrm{z}^{q\frac{1-k}{k}} \frac{\mathrm{I}_q(\mathrm{z})}{\mathrm{I}_0(\mathrm{z})} \quad (9)$$

with

$$\mathrm{I}_q(\mathrm{z}) = \int_{\mathrm{z}^{\frac{k-1}{k}}}^{\infty} dx\, x^{q+1} \sum_{m_1, m_2 \geq 1} \exp\left(-x^k \mathrm{m}_1^{k-1} \mathrm{m}_2^{2k-2} \right)$$

Having fully imparted into the critical hadronic system the physics of the underlying microscopic theory, more accurately of its universality class representative, we can proceed to derive concrete results concerning its behaviour. In what follows we furnish a number of specific predictions pertaining to the critical hadronic system limiting our arguments to the basic essentials. A more detailed analysis will be presented elsewhere [11].

a) Multidimensional intermittency patterns associated with chiral QCD phase transition. Equation (9) leads to the power laws:

$$\langle \mathrm{n}^q; a, b \rangle \approx \frac{2^q \mathrm{I}_q(0)}{\mathrm{I}_0(0)} \left(\frac{\pi^{k-1}}{c_A g_c} \right)^{\frac{q}{k}} \times \left(\frac{\Delta}{a} \right)^{q\frac{k-1}{k}} \left(\frac{R_{\perp,c}}{b\beta_c} \right)^{\frac{2q(k-1)}{k}} \quad (10)$$

valid for $\mathrm{V}_{ab} \gg \mathrm{V}_0$. The above equation shows that the distribution of hadrons of a local source at $T = T_c$ has a monofractal structure [10] with dimension $\mathrm{d}_F = 3\left(1 - \frac{1}{k}\right)$ which, due to the cylindrical geometry of the collision, is the Cartesian product of two fractals, $\mathrm{F} = \mathrm{F}_1 \times \mathrm{F}_2$, one in rapidity ($\mathrm{F}_1$) with dimension $\mathrm{d}_F^{(1)} = \frac{k-1}{k}$ and the other in transverse space (F_2) with dimension $\mathrm{d}_F^{(2)} = 2\left(1 - \frac{1}{k}\right)$.

The fractal structure in configuration space, $\mathrm{F} = \mathrm{F}_1 \times \mathrm{F}_2$, leads to a similar fractal, $\tilde{\mathrm{F}} = \tilde{\mathrm{F}}_1 \times \tilde{\mathrm{F}}_2$, in 3-d momentum space (rapidity×transverse momentum space) with $\tilde{\mathrm{F}}_1 = \mathrm{F}_1$ and $\tilde{\mathrm{F}}_2$ given by the Fourier transform of F_2. The fractal dimension of $\tilde{\mathrm{F}}_2$ is $\tilde{\mathrm{d}}_F^{(2)} = 2 - \mathrm{d}_F^{(2)}$, leading to strong intermittency in the transverse momentum plane $\left(\tilde{\mathrm{d}}_F^{(2)} = \frac{2}{k}\right)$. The projections of $\tilde{\mathrm{F}}^{(2)}$ onto its 1-d subspaces are also fractals with dimension $\frac{2}{k}$. In short, the theory predicts a complete multidimensional intermittency pattern associated with a local hadronization source which, at the level of second order factorial moments and in terms of the usual variables in momentum space (rapidity y, transverse momentum $\mathrm{q}_\perp$ and azimuthal angle φ), reads as follows

$$\mathrm{F}_2^{(1)}(\delta\varphi) \sim \mathrm{F}_2^{(1)}(\delta \mathrm{q}_\perp) \sim \mathrm{M}^{\frac{1-2\eta}{3}} ,$$

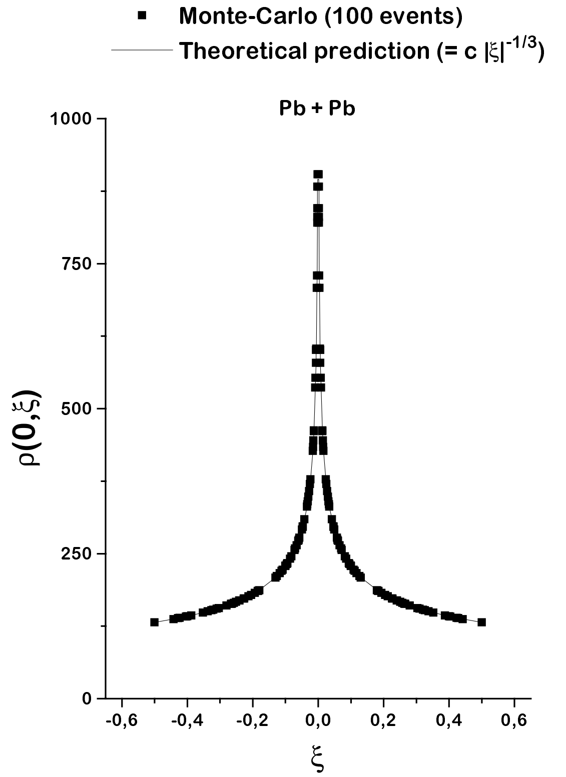

Fig. 1. The density-density correlation function $<\rho(0)\rho(\xi)>$ produced by Monte Carlo simulation of the partition function (7) using the Metropolis algorithm (*full squares*). For illustration we show in the same plot the theoretical prediction $<\rho(0)\rho(\xi)>\sim|\xi|^{-\frac{1}{k}}$ (*line*)

$$\begin{aligned}
\mathrm{F}_2^{(1)}(\delta\mathrm{y}) &\sim \mathrm{M}^{\frac{1+\eta}{3}} \\
\mathrm{F}_2^{(2)}(\delta\varphi,\delta\mathrm{y}) &\sim \mathrm{F}_2^{(2)}(\delta\mathrm{q}_\perp,\delta\mathrm{y}) \sim \mathrm{M}^{\frac{2-\eta}{3}}\ , \\
\mathrm{F}_2^{(2)}(\delta\varphi,\delta\mathrm{q}_\perp) &\sim \mathrm{M}^{\frac{4-2\eta}{3}}\ , \\
\mathrm{F}_2^{(3)}(\delta\mathrm{y},\delta\varphi,\delta\mathrm{q}_\perp) &\sim \mathrm{M}^{\frac{5-\eta}{3}}\ ,
\end{aligned} \tag{11}$$

where $\mathrm{M}^{-1}=\dfrac{\delta\mathrm{y}}{\Delta},\ \dfrac{\delta\varphi}{2\pi}\ ,\ \dfrac{\delta\mathrm{q}_\perp}{\mathrm{q}_\perp^{(0)}}$.

The power laws (11) are expressed in terms of the O(4) anomalous dimension η ($\approx$0.034) in order to emphasize the idea that the expected universality in intermittency patterns is intimately related to the chiral QCD phase transition.

b) Intermittency breakdown scale. 1-d intermittency effects, associated with the fractal F_1, break down at the level of a minimal scale in rapidity $\delta_0=\dfrac{\mathrm{V}_0}{\pi\mathrm{R}_{\perp,c}^2}$ which, in terms of the basic parameters involved, reads as follows

$$\delta_0=\frac{\beta_c^2}{\pi\mathrm{R}_{\perp,c}^2}\left(\frac{c_A g_c}{2^k}\right)^{\frac{1}{k-1}}. \tag{12}$$

The above equation suggests an upper bound $\delta_0 < \dfrac{2\mathrm{A}^{-\frac{5}{6}}}{\pi\sqrt{3}}\left(\dfrac{\beta_c}{\mathrm{R}_0}\right)^2$ which for heavy ions ($\mathrm{A}>100$) guarantees a genuine intermittency effect in rapidity for a wide range of scales ($\delta_0 < 4\times10^{-3}$).

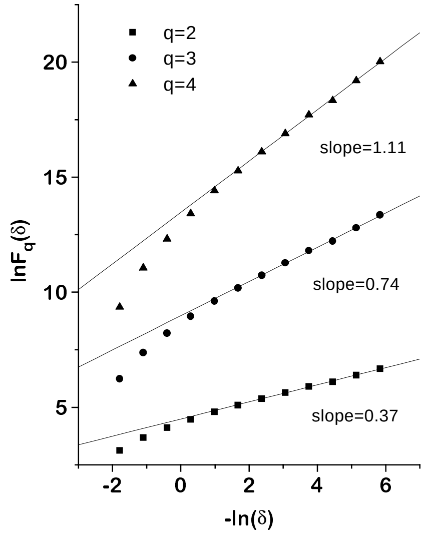

Fig. 2. The moments $F_q(\delta)=<n^{q-1};\delta>$ produced by the same Monte Carlo events as in Fig. 1 for $q=2,3,4$

c) Universal power-law for the average multiplicity. On the basis of (9), (10) and (12), the average multiplicity $\langle \mathrm{n}\rangle$ of the local system can be written in the following compact form:

$$\langle\mathrm{n}\rangle=\frac{(k-2)(3k-4)\Gamma\left(\frac{3}{k}\right)}{(2k-3)(5k-6)\Gamma\left(\frac{2}{k}\right)}\left(\frac{\Delta}{\delta_0}\right)^{\frac{k-1}{k}}. \tag{13}$$

This is a universal power-law, which depends on the critical exponent k (≈ 3), whereas the A-dependence has been absorbed in the rapidity scale δ_0. Equation (13) shows that, in critical events, there is a tendency for low multiplicities of hadrons, emitted from each local source, unless the minimal scale δ_0 is extremely small. This tendency can be understood physically as the result of a static transition of second order in which the change from quark to hadron matter occurs at once at $T=T_c$. In the absence of a mixed phase during this transition there is no chance for a cumulative hadronization effect which could enhance drastically the hadronic multiplicity.

d) Universal power-law for the density-density correlation in rapidity. The fractal dimension in rapidity space $\mathrm{d}_F^{(1)}=\frac{k-1}{k}$ implies a power-law for the corresponding density-density correlation function $<\rho(0)\rho(\xi)> = \int d^2\mathbf{x}_\perp <\phi^2(\mathbf{x}_\perp,\xi)>$. In the limit $\delta_0 \ll |\xi|$ we have $<\rho(0)\rho(\xi)>\sim\xi^{-\frac{1}{k}}$, a strong effect which is expected to appear in critical events together with 1-d intermittency (in rapidity). Using the Metropolis algorithm to generate events distributed according to the partition function (7) in (n,m_1)-space we can simulate the density-density correlation function $\rho(0,\xi)\equiv<\rho(0)\rho(\xi)>$ for Pb+Pb collisions ($A=208, \Delta=1, \delta_0=0.004$). The result is presented in Fig. 1 and the validity of this power-law is illustrated. In Fig. 2 the 1-d moments $<n^{q-1};\delta>$ for the same Monte-Carlo events are shown and the monofractal structure in rapidity (linear spectrum of intermittency indices) is illustrated. The predictions of the theory depend on two parameters, the universal index κ reflecting the critical behaviour and the nonuniversal scale in rapidity δ_0 reflecting the transverse size ($\delta_0 \sim R_\perp^{-2}$) as well as the proper-time scale τ ($\delta_0 \sim \tau^{1/\kappa-1}$). Although our study has been performed at the critical point ($T = T_c,\ \tau = \tau_c$) we claim that the universal characteristics of the system (self-similar patterns) remain valid near the freeze-out point ($T \approx T_f,\ \tau \approx \tau_f$) at which hadrons reach their free-streaming asymptotic state. In general, the distribution of hadrons, when they decouple from the interaction region, is fixed by the freeze-out temperature T_f, the corresponding time-scale τ_f and the size of the system ($\Delta, R_{\perp,f}$). For a second-order transition we have $T_f \approx T_c$ and, therefore, the effective potential (2) and the associated self-similar behaviour (10) remain valid near the freeze-out temperature. On the contrary, the time-scale τ and the transverse radius $R_\perp$ are likely to change drastically ($\tau_f \gg \tau_c,\ R_{\perp,f} \gg R_{\perp,c}$) affecting, through the parameter $\delta_0 \sim \tau^{1/\kappa-1} R_\perp^{-2}$, the nonuniversal properties of the hadronic distribution. As a result, the average multiplicity $< n >$ per local hadronization source (13) has to be modified by a factor $< n > \sim \left(\frac{R_{\perp,f}}{R_{\perp,c}}\right)^{\frac{2(\kappa-1)}{\kappa}} \left(\frac{\tau_c}{\tau_f}\right)^{\frac{1}{\kappa}}$ corresponding to the transition from the critical to freeze-out time-scale. This factor gives a measure of the multiplicity change during the life-time of the critical system.

4 Conclusions

A number of characteristic properties of the critical hadronic system, viewed as a local hadronization source in thermodynamic equilibrium centered at the position (in rapidity space) of an inertial observer $\xi = \xi_o$, are dictated by the anomalous dimension η of the $O(4)$ Heisenberg model. These properties underlie the structure of the relevant critical events in ultrarelativistic nuclear collisions and *can* be revealed by employing an event by event analysis. To determine the exact structure of these events requires however the extension of our local model to a global one consisting of several, non overlapping, hadronization sources with centers distributed randomly in the whole available space. In particular it is interesting to study the influence of the existense of more than one sources to the fractal character of the hadronic system and consequently to the intermittency pattern of the corresponding factorial moments. A complete treatment of this case leading to the generation of observable critical events and their analysis is still in progress [11].

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References

1. T.D. Lee, Proc. Quark Matter '95, Nucl. Phys. **A590** 11c (1995)
2. F. Wilczek, Int. J. Mod. Phys. **A7**, 3911 (1992)
3. N.G. Antoniou, Proc. Cracow Workshop on Multiparticle Production 1993, Eds. A. Bialas, K. Fialkowski, K. Zalewski, R.C. Hwa, World Scientific, Singapore 1994, p. 187
4. H.-Th. Elze, Nucl. Phys. **A566**, 571c (1994)
5. J.D. Bjorken, Phys. Rev. **D27**, 140 (1983)
6. M. Reuter, N. Tetradis, C. Wetterich, Nucl. Phys. **B401**, 567 (1993); N. Tetradis, C. Wetterich, Nucl. Phys. **B422**, 541 (1994)
7. N. Tetradis, private communication
8. D.J. Scalapino, R.L. Sugar, Phys. Rev. **D8**, 2284 (1973); J.C. Botke, D.J. Scalapino, R.L. Sugar, Phys. Rev. **D9**, 813 (1974)
9. N.G. Antoniou, A.I. Karanikas, S.D.P. Vlassopulos, Phys. Rev. **D29**, 1470 (1984); P. Carruthers, I. Sarcevic, Phys. Lett. **B189**, 442 (1987); N.G. Antoniou, A.I. Karanikas, C.G. Papadopoulos, Phys. Lett. **B233**, 423 (1989); H.-Th. Elze, I. Sarcevic, Phys. Rev. Lett. **68**, 1988 (1992); R.C. Hwa, M.T. Nazirov, Phys. Rev. Lett. **69**, 741 (1992); I.M. Dremin, M.T. Nazirov, Z. Phys. **C59**, 647 (1 A.K. Mohanty, S.K. Kataria, Phys. Rev. Lett. **73**, 2672 (1994); H.C. Eggers, H-Th. Elze, I. Sarcevic, Int.J. Mod. Phys. **A9**, 3821 (1994); N.G. Antoniou, C.N. Ktorides, I.S. Mistakidis, F.K. Diakonos, Phys. Lett. **B353**, 501 (1995)
10. H. Satz, Nucl. Phys. **B326**, 613 (1989)
11. N.G. Antoniou, F.K. Diakonos, C.N. Ktorides, M. Lahanas, in preparation